Published by Institute of Physics Publishing for SISSA/ISAS
Received: *January 10, 2004*
Accepted: *April 6, 2004*

Large-N duality, Lens spaces and the Chern-Simons matrix model

Nick Halmagyi,[ab] **Takuya Okuda**[c] **and Vadim Yasnov**[a]

[a] *Department of Physics and Astronomy, University of Southern California Los Angeles, CA 90089, U.S.A.*
[b] *Kavli Institute for Theoretical Physics, University of California Santa Barbara, CA 93106, U.S.A.*
[c] *California Institute of Technology Pasadena, CA, 91125, U.S.A.*
E-mail: halmagyi@physics.usc.edu, takuya@theory.caltech.edu, yasnov@physics.usc.edu

Abstract: We demonsrate that the spectral curve of the matrix model for Chern-Simons theory on the Lens space $S^3/\mathbb{Z}_p$ is precisely the Riemann surface which appears in the mirror to the blownup, orbifolded conifold. This provides the first check of the A-model large-N duality for $T^*(S^3/\mathbb{Z}_p)$, $p > 2$.

Keywords: 1/N Expansion, Chern-Simons Theories, Matrix Models.

Contents

1. Introduction

The conifold transition is an example of an open/closed string duality. In the topological A-model, this is a duality between the open A-model on $T^*(S^3)$ (which is equivalent to Chern-Simons (CS) theory on S^3 [1]), and Kahler gravity on $\mathcal{O}_{-1}+\mathcal{O}_{-1}\to\mathbb{P}^1$. This was originally studied by taking the partition function of large-N Chern-Simons (CS) theory on S^3 expanded in a 't Hooft limit and presenting it first in the form of an open string theory [2] (i.e. an expansion in genus and holes) and then summing over the holes to get a closed string theory [3]. A worldsheet proof of this duality has since been provided [4].

It is of some interest to extend this to a duality between $T^*(S^3/\mathbb{Z}_p)$ and a $\mathbb{Z}_p$ orbifold of the resolved conifold. Whilst an explicit form of the partition function for CS theory on $S^3/\mathbb{Z}_p$ is known [5], this form includes the summation over all vacua in the theory, yet for the purposes of string theory we want only the contribution from a single vacuum. For perhaps these reasons it has not been possible to exhibit the large-N duality of $T^*(S^3/\mathbb{Z}_p)$ at the level of partition function however the worldsheet proof does generalize to $T^*(S^3/\mathbb{Z}_p)$ and other geometries [6].

In [7, 8] it was shown that CS theory on $S^3/\mathbb{Z}_p$ has a matrix model description. In [8] it was also shown that Holomorphic Chern-Simons (HCS) theory reduced to $\mathbb{P}^1$'s inside the mirror (call it $\widetilde{X}$) to $T^*(S^3/\mathbb{Z}_p)$ has a matrix model description. Further, these matrix models are identical. For the case $p=2$ the partition function of this matrix model was calculated perturbatively and was shown to agree with the Kodaira-Spencer theory [9] predictions from the large-N dual geometry, providing solid evidence for the proposed duality. For CS theory on S^3 the matrix model was solved to all genus using orthogonal polynomials in [10] and the orientifold of the conifold was studied in [11].

The manifold $\widetilde{X}$ is given by the blowup of

$$xy = F_p(e^u, e^v)\,, \tag{1.1}$$

where

$$F_p(e^u, e^v) = (e^v-1)(e^{v+pu}-1) \tag{1.2}$$

and by the general arguments of Dijkgraaf-Vafa theory [12, 13], the spectral curve of the corresponding matrix model should be a complex structure deformation of $F_p=0$. In [14] two of the current authors found an expression for the spectral curve of the matrix model for CS theory on $S^3/\mathbb{Z}_p$. This involved first showing that although this matrix model looks similar to a p-matrix model, it does in fact have square root branch cuts, therefore its spectral curve has only two sheets. This led to an explicit expression for the resolvent, depending on $p-1$ parameters d_i which in principle could be found perturbatively by performing the A-cycle integrals. The spectral curve can be read off from the resolvent and the d_i correspond to complex structure moduli. This will be reviewed in section 2.

In section 3 we use toric geometry to construct a resolution of the $\mathbb{Z}_p$ orbifold of the resolved conifold. This is a particular A_p fibration over $\mathbb{P}^1$. Then using the Hori-Vafa mirror map we can write down the mirror geometry and find that after a suitable coordinate redefinition, the non-trivial Riemann surface inside this threefold is precisely the spectral curve found in [14]. This explicitly identifies the large-N dual of $T^*(S^3/\mathbb{Z}_p)$ for all $p>1$. The matching of the geometries proves the equivalence of the leading order (in g_s) free energy between the matrix model and the closed A-model on this particular fibration. To our knowledge this is the first check of this large-N duality for $p>2$.

2. The matrix model spectral curve

The partition function of CS theory on the Lens space $S^3/\mathbb{Z}_p$ [8] can be written as a matrix integral over p sets of eigenvalues, which we label by an index $I \in \{0,..,p-1\}$. The Ith set contains N_I eigenvalues. The measure is a product of two factors, a self interaction term (Δ_1) and a term containing the interaction between different sets of eigenvalues (Δ_2),

$$\Delta_1(u) = \prod_I \prod_{i\neq j} \left(2\sinh\left(\frac{u_i^I - u_j^I}{2}\right)\right)^2 \tag{2.1}$$

$$\Delta_2(u) = \prod_{I<J} \prod_{i,j} \left(2\sinh\left(\frac{u_i^I - u_j^J + d^{IJ}}{2}\right)\right)^2, \tag{2.2}$$

where $d^{IJ} = 2\pi i(I-J)/p$. The potential has an overall factor of p compared to the S^3 case,

$$V(u) = p\sum_{I,i} \frac{\left(u_i^I\right)^2}{2}. \tag{2.3}$$

In the above notations the CS partition function becomes

$$Z \sim \int \prod_{I=0}^{p-1} \prod_{i=1}^{N_I} du_i^I \Delta_1(u)\Delta_2(u)\exp\left(-\frac{1}{g_s}V(u)\right). \tag{2.4}$$

We define individual resolvents for each set of the eigenvalues by

$$\omega_I(z) = g_s \sum_i \coth\left(\frac{z-u_i^I}{2}\right) \tag{2.5}$$

and the total resolvent, which we are most interested in is

$$\omega(z) = \sum_I \omega_I \left(z - \frac{2\pi i I}{p} \right) . \tag{2.6}$$

Anticipating taking the large-N limit we also introduce 't Hooft parameters $S_I = g_s N_I$ and $S = \sum_I S_I$. The equation of motion for each eigenvalue is

$$p u_i^I = g_s \sum_{j \neq i} \coth \left(\frac{u_i^I - u_j^I}{2} \right) + g_s \sum_{J \neq I} \sum_j \coth \left(\frac{u_i^I - u_j^J + d^{IJ}}{2} \right) . \tag{2.7}$$

From the large-N limit of this equation we can derive

$$\frac{1}{2}\omega^2(z) - p \sum_I \left(z - \frac{2\pi i I}{p} \right) \omega_I \left(z - \frac{2\pi i I}{p} \right) = f(z) , \tag{2.8}$$

where $f(z)$ is a regular function

$$f(z) = p\ g_s \sum_I \sum_i \left(u_i^I + \frac{2\pi i I}{p} - z \right) \coth \left(\frac{z - u_i^I - 2\pi i I/p}{2} \right) + \frac{S^2}{2} . \tag{2.9}$$

Given the large-N limit of the equations of motion it is possible to find the total resolvent $\omega(z)$. This method has been developed and checked in [14] which we will now review. We assume that the eigenvalues spread only along the real line. For general multi matrix models this is not true [15, 16, 17]. However, this assumption leads to the correct result for our case. Note that we do not make any assumption on the type of the cuts. In the total resolvent $\omega(z)$, the individual resolvents come with relative shifts of the argument by $2\pi i I/p$. Therefore the cuts in the total resolvent are now separated by $2\pi i I/p$. For example, if $\omega_0(z)$ jumps at the point z, all other individual resolvents $\omega_I(z - 2\pi i I/p)$ with $I \neq 0$ are regular at this point. This means that on Ith cut the total resolvent jumps only due to the resolvent $\omega_I(z)$. From this it follows that

$$\frac{1}{2} \left(\omega_+ \left(z + \frac{2\pi i I}{p} \right) + \omega_- \left(z + \frac{2\pi i I}{p} \right) \right) = pz, \quad (I'\text{th cut}) \tag{2.10}$$

and so every cut is a square root.

We label the contour around the Ith cut as A_I. From (2.6) it is clear that

$$\lim_{z \to \infty} \omega(z) \ = \ S , \tag{2.11}$$

$$\lim_{z \to -\infty} \tilde{\omega}(z) \ = \ -S , \tag{2.12}$$

where $\tilde{\omega}(z)$ is the value of the resolvent on the second sheet. From (2.5) we also have that

$$\frac{1}{2} \oint_{A_I} \omega(z) dz = 2\pi i S_I . \tag{2.13}$$

Since the integral over the $A = \sum_I A_I$ cycle is fixed by (2.12), there are only $p-1$ independent periods. Now we construct a regular function,

$$g(Z) = e^{\omega/2} + Z^p e^{-\omega/2} , \tag{2.14}$$

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which has the limiting behavior,

$$\lim_{Z\to\infty} g(Z) = e^{-S/2} Z^p \tag{2.15}$$

$$\lim_{Z\to 0} g(Z) = e^{-S/2} \tag{2.16}$$

and is thus of the form,

$$g(Z) = e^{-S/2}(Z^p + d_{p-1} Z^{p-1} + \cdots + d_1 Z + 1)\,. \tag{2.17}$$

The function $g(Z)$ depends on $p-1$ moduli d_n, which could be found by evaluating the period integrals (2.13). Since we have already fixed the integral over the cycle $A = \sum_I A_I$ by (2.16), there are only $p-1$ independent A-periods.

We can solve (2.14) for $\omega(Z)$ to get

$$\frac{\omega(Z)}{2} = \log\left(\frac{1}{2}\left(g(Z) - \sqrt{g^2(Z) - 4Z^p}\right)\right), \tag{2.18}$$

the function under the square root is a polynomial of the degree $2p$, it has $2p$ distinct roots that depend on only $p-1$ parameters. Thus the spectral curve consists of two cylinders glued together along p cuts. Note that the center of the I'th cut is at the point $z = 2\pi i I/p$. The cycles around each cut are A cycles. There is also a set of noncompact cycles, the B cycles. Each B_I cycle starts at infinity on the physical sheet (the sheet where the resolvent is finite) and goes through the I'th cut to a point at infinity on the other sheet.

From (2.18) we see that the spectral curve is given by

$$(e^v - 1)(e^{pu+v} - 1) + e^S - 1 + e^v \sum_{n=1}^{p-1} d_n e^{nu} = 0\,, \tag{2.19}$$

which is a complex structure deformation of $F_p = 0$ (from (1.1)). Here $z \equiv u$ and $v = (S-\omega)/2$. It is worth mentioning that the functions $d_n(S_I)$ are available as a power series in 't Hooft parameters S_I's which is valid only in the region of small S_I. We will see that this region corresponds to negatively large values of Kahler parameters in the A-model.

3. The orbifold of the resolved conifold

We will now construct the $\mathbb{Z}_p$ orbifold of the resolved conifold ($\mathcal{O}_{-1} + \mathcal{O}_{-1} \to \mathbb{P}^1$) using standard toric methods.[1] We will then apply the Hori-Vafa mirror map, from which we can obtain the Riemann surface which is the non-trivial part of the mirror geometry.

The deformed conifold ($T^*(S^3)$) can be written as $\det A = \mu$, where

$$A = \begin{pmatrix} z_1 & z_3 \\ z_2 & z_4 \end{pmatrix}. \tag{3.1}$$

[1] a good reference for the basics of toric geometry is [18].

We can orbifold this geometry by a $\mathbb{Z}_p$ symmetry generated by

$$A \mapsto \begin{pmatrix} \alpha & 0 \\ 0 & \alpha^{-1} \end{pmatrix} A, \tag{3.2}$$

where $\alpha = e^{2\pi i/p}$. The S^3 is given by $|z_1|^2 + |z_3|^2 = \mu$ (assuming μ is real and positive) and the $\mathbb{Z}_p$ acts on this as $(z_1, z_3) \to (\alpha z_1, \alpha z_3)$, which gives the Lens space $S^3/\mathbb{Z}_p = L(p, 1)$, thus the entire threefold is now $T^*(S^3/\mathbb{Z}_p)$.

We now perform this orbifold action on the other side of the proposed large-N duality. The resolved conifold can be expressed as

$$A \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0\,, \tag{3.3}$$

where $(\lambda_1 : \lambda_2)$ are the homogeneous coordinates on $\mathbb{P}^1$. We simply extend the $\mathbb{Z}_p$ action given by (3.2) to the resolved conifold, clearly this will act trivially on the base $\mathbb{P}^1$ but non-trivially on the fibre co-ordinates since these are contained in the matrix A. Thus the resulting orbifold is a particular fibration of an A_p-singularity over $\mathbb{P}^1$. We will show that the large-N dual of $T^*(S^3/\mathbb{Z}_p)$ is the blowup of this space.

The procedure of blowing up can be convenient described in the language of toric geometry but first we need the fan for the singular manifold. The standard two coordinate patches of $\mathbb{P}^1$ can be used to trivialize the $\mathcal{O}_{-1} + \mathcal{O}_{-1} \to \mathbb{P}^1$ bundle. On the first patch we have coordinates $(u, x_1, x_2) = (\lambda_1/\lambda_2, z_1, z_2)$ and on the second we have $(v, y_1, y_2) = (\lambda_2/\lambda_1, -z_3, -z_4)$. On the overlap of the two patches these coordinates are related by $v = 1/u, y_1 = ux_1, y_2 = ux_2$. $\mathbb{Z}_p$ acts on the fiber coordinates. For the quotient of the first patch, therefore, we introduce invariant coordinates $(\xi_1, \xi_2, \xi_3) := (x_1^p, x_2^p, x_1x_2)$ satisfying $\xi_1\xi_2 = \xi_3^p$. Similarly we introduce (η_1, η_2, η_3) satisfying $\eta_1\eta_2 = \eta_3^p$ for the second patch. These two sets of coordinates are glued togther by $v = 1/u, \eta_1 = u^p\xi_1, \eta_2 = u^p\xi_2, \eta_3 = u^2\xi_3$.

Toric geometry is designed to encode these transition functions (see the appendix for notations). Namely, we get the relations

$$\begin{aligned} u_{12} + u_{13} &= pu_{14}\,, & u_{22} + u_{23} &= pu_{24}\,, \\ u_{21} &= -u_{11}\,, & u_{22} &= pu_{11} + u_{12}\,, \\ u_{23} &= pu_{11} + u_{13}\,, & u_{24} &= 2u_{11} + u_{14} \end{aligned} \tag{3.4}$$

for the lattice vectors in M. The first two relations imply that u_{14} and u_{24} are not vertices of the dual cones $\check{\sigma}_1$ and $\check{\sigma}_2$ respectively, where $\check{\sigma}_1$ is spanned by u_{11}, u_{12}, u_{13}, and $\check{\sigma}_2$ is spanned by u_{21}, u_{22}, u_{23}. We choose a basis such that $u_{11} = (1, 0, 0), u_{12} = (0, 1, 0), u_{14} = (0, 0, 1)$, this determines all remaining vectors u_{ij}.

From this we find that the cone σ_1 is generated by v_1, v_2, v_3, and the cone σ_2 is generated by v_4, v_2, v_3, where $v_1 = (1, 0, 0), v_2 = (0, p, 1), v_3 = (0, 0, 1), v_4 = (-1, p, 2)$. After a $GL(3, \mathbb{Z})$ transformation $(x', y', z') = (x + y, x, x + z)$, they become $v_i = (\nu_i, 1)$, where $\nu_1 = (1, 1), \nu_2 = (p, 0), \nu_3 = (0, 0), \nu_4 = (p - 1, -1)$. The toric fan of $A_p \to \mathbb{P}^1$ consists of the cones σ_1, σ_2 and their faces.

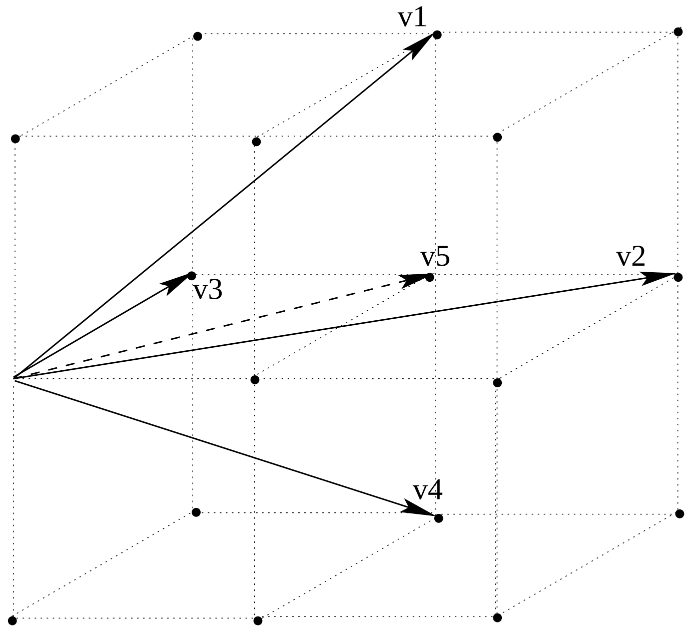

Figure 1: The fan for the resolution of the $\mathbb{Z}_2$ orbifold of $\mathcal{O}_{-1} + \mathcal{O}_{-1} \to \mathbb{P}^1$.

To obtain a smooth manifold, we subdivide the cones σ_1, σ_2 by introducing $v_i = (\nu_i, 1)$ where $\nu_i = (i-4, 0)$ for $i = 5, \ldots, p+3$. The fan Δ of the smooth toric manifold is the union of $[v_1, v_3, v_5]$, $[v_1, v_{p+3}, v_2]$, $[v_1, v_5, v_6]$, $[v_1, v_6, v_7]$, …, $[v_1, v_{p+2}, v_{p+3}]$ from σ_1 and $[v_4, v_3, v_5]$, $[v_4, v_{p+3}, v_2]$, $[v_4, v_5, v_6]$, $[v_4, v_6, v_7]$, …, $[v_4, v_{p+2}, v_{p+3}]$ from σ_2 (see figure 1). Here $[u, v, w]$ denotes the cone spanned by the vectors u, v, w. It is easily seen that each cone now has volume equal to 1, where we normalize the volume of the cone generated by $[1,0,0], [0,1,0], [0,0,1]$ to 1. The toric web diagrams for these fans are drawn for the cases $p = 1, 2, 3$ in figure 2. This geometry and other A_p fibrations over $\mathbb{P}^1$ were recently considered in [20, 21] for the purposes of geometric engineering [22]. Setting $m = 0$ in [21] gives the above geometry.

From the data of the fan Δ we can read off the charge vectors $Q_a, a = 0, 1, \cdots, p-1$ of the corresponding linear sigma model. They are the generators of the lattice $D = \{Q \in \mathbb{Z}^{p+3} | \sum_i Q_i v_i = 0\}$ [19]. After rearranging the columns, these are given by

$$\begin{aligned} Q_0 &= (0, 1, 1, -1, -1, 0, \ldots, 0, 0, 0, \ldots, 0)\,, \\ Q_j &= \left(-(j+1), j, 0, 0, 0, 0, \ldots, 0, \overset{(5+j)\text{th}}{1}, 0, \ldots 0\right), \\ Q_{p-1} &= (-p, p-1, 1, 0, 0, 0, \ldots, 0, 0, 0, \ldots, 0)\,, \end{aligned} \tag{3.5}$$

where $1 \leq j \leq p-2$. According to [23, 24], the mirror geometry is given by $zw = \sum_{i=0}^{p+2} e^{-y_i}$ where the fields are related by $\sum_i Q_{ai} y_i = t_a$ and one of the y_i variables is to be set to zero. Eliminating y_2, y_4, y_{4+j} $(1 \leq j \leq p-2)$, and setting $y_1 = 0$, we get the Riemann surface inside this threefold to be

$$0 = 1 + e^{-y_0} + e^{-y_3} + e^{-t_{p-1}} e^{-py_0} + e^{t_0 - t_{p-1}} e^{-py_0 + y_3} + \sum_{j=1}^{p-2} e^{-t_j} e^{-(j+1)y_0}\,. \tag{3.6}$$

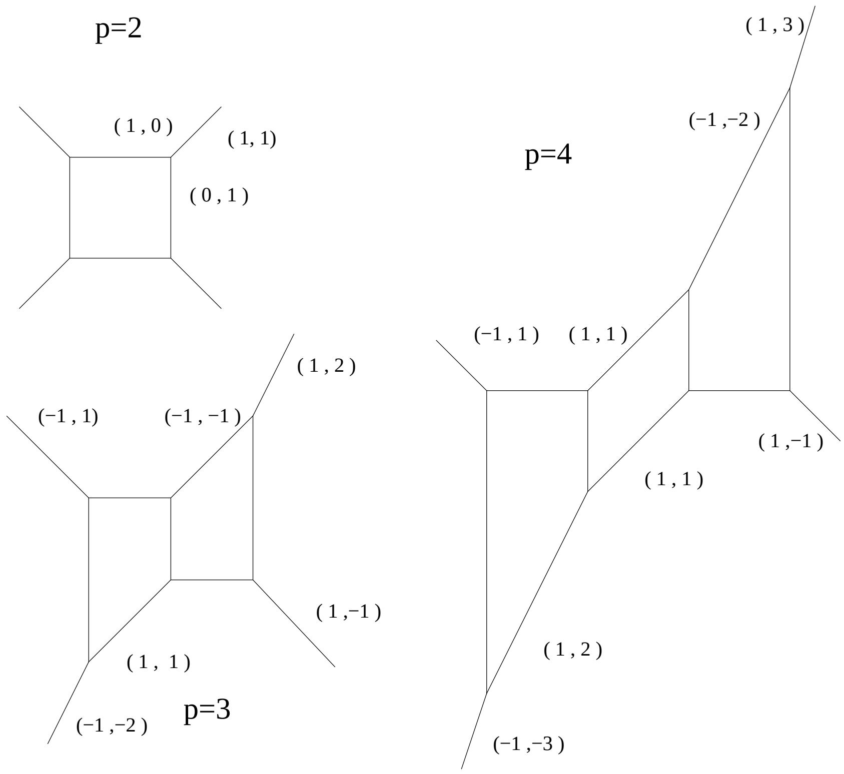

Figure 2: The toric web diagrams of the large-N dual of $T^*(S^3/\mathbb{Z}_p)$ for some values of p.

Now after the co-ordinate transformation $u = -y_0 - t_{p-1}/p, v = y_3 + t_0 + \pi i$, we can write this as

$$(e^v - 1)(e^{pu+v} - 1) + e^{t_0} - 1 - e^v \left(e^{t_{p-1}/p} e^u + \sum_{j=1}^{p-2} e^{-t_j + (j+1)t_{p-1}/p} e^{(j+1)u} \right) = 0 \qquad (3.7)$$

which is precisely (2.19).

Acknowledgments

NH would like to thank Jaume Gomis for useful discussions. TO would like to thank Hiroshi Ooguri for collaboration on a related project [6]. We also thank Hiroshi Ooguri for reading the manuscript. This research was supported in part (NH) by the National Science Foundation under Grant No. PHY99-07949 and (TO) by DOE grant DE-FG03-92ER40701.

A. Toric variety described by a fan

A fan Δ in $N = \mathbb{Z}^n$ is a collection of strongly convex rational polyhedral cones in $N_{\mathbb{R}} = N \otimes_{\mathbb{Z}} \mathbb{R}$, such that (i) a face of any cone in Δ is also a cone in Δ and (ii) the intersection of two cones in Δ is a face of each cone.

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The procedure to construct the charts and transition functions is the following [18].

1. Let M be the dual lattice of N. For each cone $\sigma_i \in \Delta$ of maximal dimension, define the dual cone as $\check{\sigma}_i := \{u \in M_{\mathbb{R}} | (u,v) \geq 0, \forall v \in \sigma_i\}$.

2. For each dual cone $\check{\sigma}_i$, choose lattice points $u_{i,j}$ $(j = 1, 2, \cdots, r_i)$ such that $\check{\sigma}_i \cap M = \mathbb{Z}_+ u_{i,1} + \cdots + \mathbb{Z}_+ u_{i,r_i}$ $(\mathbb{Z}_+ = \{0, 1, 2, \cdots\})$.

3. For each dual cone $\check{\sigma}_i$, find a set of fundamental relations of $u_{i,1}, \cdots, u_{i,r_i}$ in the form $\sum_{j=1}^{r_i} p_{s,j} u_{i,j} = 0$, $s = 1, \cdots, R_i$.

4. For each cone σ_i, define the patch as $U_{\sigma_i} := \{(z_{i,1}, \cdots, z_{i,r_i}) \in \mathbb{C}^{r_i} | z_{i,1}^{p_{s,1}} \cdots z_{i,r_i}^{p_{s,r_i}} = 1, \forall s\}$.

5. For each pair of cones σ_i, σ_j, find a set of fundamental relations of $u_{i,1}, \ldots, u_{i,r_i}, u_{j,1}, \ldots, u_{j,r_j}$ in the form $\sum_{l=1}^{r_i} q_{i,j,l} u_{i,l} + \sum_{l=1}^{r_j} q'_{i,j,l} u_{j,l} = 0$.

6. Glue the two patches via $z_{i,1}^{q_{i,j,1}} \cdots z_{i,r_i}^{q_{i,j,r_i}} z_{j,1}^{q'_{i,j,1}} \cdots z_{j,r_j}^{q'_{i,j,r_j}} = 1$.

References

[1] E. Witten, *Chern-Simons gauge theory as a string theory*, *Prog. Math.* **133** (1995) 637 [hep-th/9207094].

[2] R. Gopakumar and C. Vafa, *M-theory and topological strings, I*, hep-th/9809187.

[3] R. Gopakumar and C. Vafa, *On the gauge theory/geometry correspondence*, *Adv. Theor. Math. Phys.* **3** (1999) 1415 [hep-th/9811131].

[4] H. Ooguri and C. Vafa, *Worldsheet derivation of a large-N duality*, *Nucl. Phys.* **B 641** (2002) 3 [hep-th/0205297].

[5] E. Witten, *Quantum field theory and the jones polynomial*, *Commun. Math. Phys.* **121** (1989) 351.

[6] T. Okuda and H. Ooguri, to appear.

[7] M. Marino, *Chern-Simons theory, matrix integrals and perturbative three-manifold invariants*, hep-th/0207096.

[8] M. Aganagic, A. Klemm, M. Marino and C. Vafa, *Matrix model as a mirror of Chern-Simons theory*, *J. High Energy Phys.* **02** (2004) 010 [hep-th/0211098].

[9] M. Bershadsky, S. Cecotti, H. Ooguri and C. Vafa, *Kodaira-Spencer theory of gravity and exact results for quantum string amplitudes*, *Commun. Math. Phys.* **165** (1994) 311 [hep-th/9309140].

[10] M. Tierz, *Soft matrix models and Chern-Simons partition functions*, hep-th/0212128.

[11] N. Halmagyi and V. Yasnov, *Chern-Simons matrix models and unoriented strings*, *J. High Energy Phys.* **02** (2004) 002 [hep-th/0305134].

[12] R. Dijkgraaf and C. Vafa, *Matrix models, topological strings and supersymmetric gauge theories*, *Nucl. Phys.* **B 644** (2002) 3 [hep-th/0206255].

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[13] R. Dijkgraaf and C. Vafa, *A perturbative window into non-perturbative physics*, hep-th/0208048.

[14] N. Halmagyi and V. Yasnov, *The spectral curve of the Lens space matrix model*, hep-th/0311117.

[15] R. Dijkgraaf and C. Vafa, *On geometry and matrix models*, *Nucl. Phys.* **B 644** (2002) 21 [hep-th/0207106].

[16] C. Hofman, *Super Yang-Mills with flavors from large-N_f matrix models*, *J. High Energy Phys.* **10** (2003) 022 [hep-th/0212095].

[17] C.I. Lazaroiu, *Holomorphic matrix models*, *J. High Energy Phys.* **05** (2003) 044 [hep-th/0303008].

[18] B.R. Greene, *String theory on Calabi-Yau manifolds*, hep-th/9702155.

[19] D.R. Morrison and M. Ronen Plesser, *Summing the instantons: quantum cohomology and mirror symmetry in toric varieties*, *Nucl. Phys.* **B 440** (1995) 279 [hep-th/9412236].

[20] A. Iqbal and A.-K. Kashani-Poor, *Instanton counting and Chern-Simons theory*, *Adv. Theor. Math. Phys.* **7** (2004) 457 [hep-th/0212279].

[21] A. Iqbal and A.-K. Kashani-Poor, SU(N) *geometries and topological string amplitudes*, hep-th/0306032.

[22] S. Katz, A. Klemm and C. Vafa, *Geometric engineering of quantum field theories*, *Nucl. Phys.* **B 497** (1997) 173 [hep-th/9609239].

[23] K. Hori and C. Vafa, *Mirror symmetry*, hep-th/0002222.

[24] M. Aganagic and C. Vafa, *Mirror symmetry, D-branes and counting holomorphic discs*, hep-th/0012041.

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